

PHYSICS

Motion, Vectors, Gravity and Projectile Motion

Grade 10 • Lessons, diagrams and fully worked examples

How to use this textbook. Read the concept lesson before each question. Identify the known quantities, draw and label a diagram, select a positive direction, choose the correct equation, and show units and direction in the final answer.

Mathematical toolkit. A vector needs both a magnitude and a direction. Use the Pythagorean theorem for perpendicular components and the sine, cosine and tangent ratios for right triangles. Most of the motion equations assume constant velocity or constant acceleration; check the assumptions first.

Notation. Δ means “change in”; the subscripts i and f indicate initial and final values. For example, v_i is initial velocity, v_f is final velocity, and Δt is elapsed time.

Lesson 1 — Distance, displacement, and compass directions

An object can travel along a route but finish much closer to its starting point. **Distance** measures the entire route and cannot be negative. **Displacement** measures only the change from start to finish and requires a direction. Picture drawing a straight arrow from the starting position to the ending position. That arrow is the displacement, even if the actual route is bent.

How the method works. For perpendicular movements, draw a right triangle: the horizontal and vertical changes are its two shorter sides. Apply the Pythagorean theorem to find the length of the diagonal, then use tangent to find the angle. Compass notation [S θ W] means start facing south and rotate θ degrees toward west.

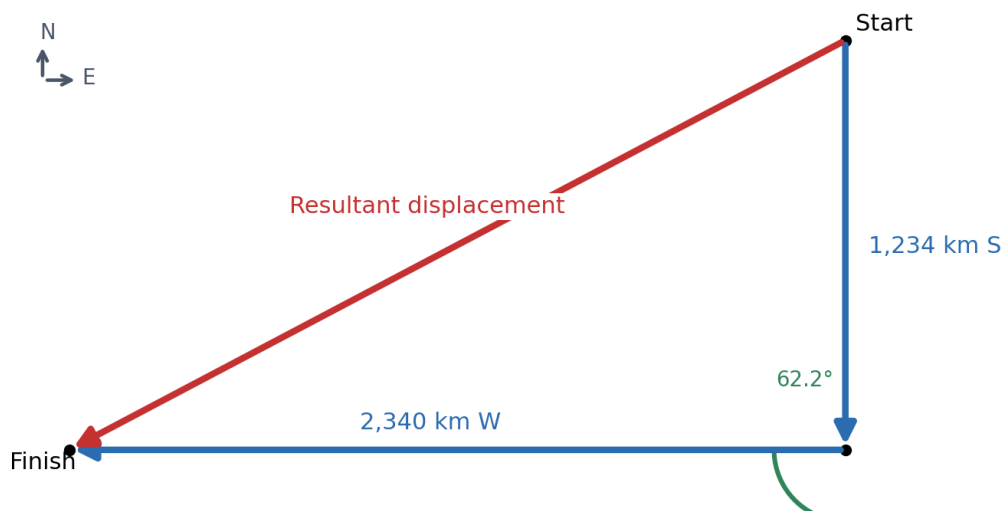
Key equations: Distance = sum of path lengths; $|\text{displacement}| = \sqrt{(\Delta x^2 + \Delta y^2)}$; $\tan \theta = \text{opposite} / \text{adjacent}$

Reasoning check. Draw the actual route in one color and the straight-line start-to-finish arrow in another. Check which compass direction the resultant really points.

Worked Example 1 — Airplane: distance and displacement

Question. A plane flies 1,234 km south, then turns and flies 2,340 km west. Determine the total distance travelled and the displacement.

Airplane displacement diagram



Step-by-step solution

- Total distance = 1,234 km + 2,340 km = 3,574 km.
- Displacement magnitude: $\Delta d = \sqrt{(1234^2 + 2340^2)} = 2,645.44$ km.
- Direction: $\tan \theta = 2340 / 1234$, so $\theta = 62.2^\circ$. This angle is measured west of south.

Final answer. Distance = 3,574 km; displacement = 2,645 km [S 62.2° W].

Lesson 2 — Combining several movements on two axes

A route with several turns may look complicated, but only its total east–west and north–south changes matter for displacement. Imagine all eastward movements as positive x , westward as negative x , northward as positive y , and southward as negative y . Combine each group before doing any trigonometry.

How the method works. The resulting horizontal and vertical totals are called the **net components**. They form a right triangle, just like a two-step trip. If west and east movements partly cancel, use the remaining westward amount, not the sum of their lengths.

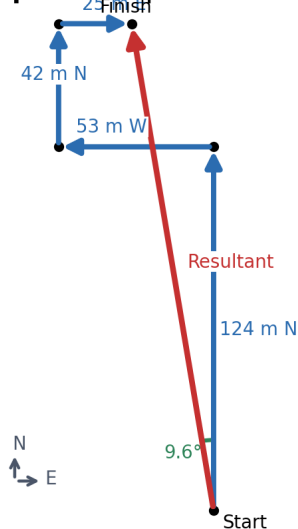
Key equations: $\Delta x = \Sigma x\text{-components}$; $\Delta y = \Sigma y\text{-components}$; $|\Delta d| = \sqrt{(\Delta x^2 + \Delta y^2)}$

Reasoning check. Add movements with signs before taking the square root. A net x -component of -28 m means 28 m west, not a negative distance.

Worked Example 2 — Squirrel: overall displacement from several moves

Question. A squirrel runs 124 m north, then 53 m west, then 42 m north, and finally 25 m east. What is its overall displacement?

Squirrel displacement diagram



Step-by-step solution

- Vertical component: 124 m north + 42 m north = 166 m north.
- Horizontal component: 53 m west – 25 m east = 28 m west.
- Displacement magnitude: $\Delta d = \sqrt{(166^2 + 28^2)} = 168.34$ m.
- Direction: $\tan \theta = 28 / 166$, so $\theta = 9.6^\circ$ west of north.

Final answer. The overall displacement is 168 m [N 9.6° W].

Lesson 3 — Breaking a diagonal vector into components

A diagonal motion can be understood as a combination of two independent movements at right angles. The vector is the diagonal of a right triangle. Its north–south part is the vertical component and its east–west part is the horizontal component. This technique is called **resolving a vector**.

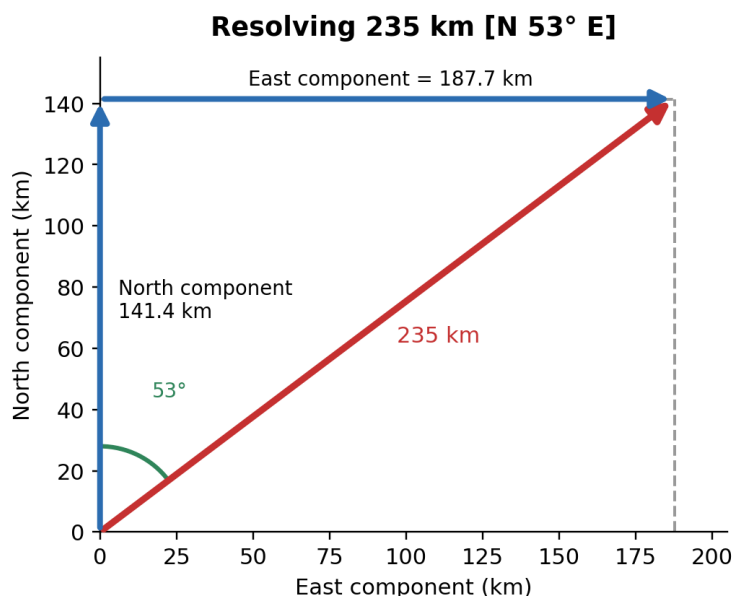
How the method works. Use sine or cosine according to where the stated angle is measured. The side touching the angle (other than the diagonal) is **adjacent** and uses cosine. The side across from the angle is **opposite** and uses sine. For an angle measured east of north, north is adjacent and east is opposite.

Key equations: $\text{Adjacent} = \text{magnitude} \times \cos \theta$; $\text{opposite} = \text{magnitude} \times \sin \theta$

Reasoning check. Sketch the angle at the origin and label the adjacent side before choosing the trigonometric function. Make sure both components have the same units as the vector.

Worked Example 3 — Goose: finding vector components

Question. A goose flies 235 km [N 53° E]. Find its north and east components.



Step-by-step solution

- North component: $\Delta y = 235 \cos 53^\circ = 141.43 \text{ km}$.
- East component: $\Delta x = 235 \sin 53^\circ = 187.68 \text{ km}$.

Final answer. North component = 141 km; east component = 188 km.

Lesson 4 — Average speed and changing units

Speed answers the question “how fast?” without giving a direction. **Average speed** uses the total distance travelled and the total elapsed time. Even if an object speeds up or slows down along the route, this formula gives the overall average. A speed of 1 km/h means 1 kilometre of travel in one hour.

How the method works. Physics often uses metres and seconds. Since $1 \text{ km} = 1,000 \text{ m}$ and $1 \text{ h} = 3,600 \text{ s}$, $1 \text{ km/h} = 1000/3600 \text{ m/s}$. Therefore divide by 3.6 to change km/h to m/s, and multiply by 3.6 to change m/s to km/h.

Key equations: Average speed = total distance / Δt ; $\text{m/s} = (\text{km/h}) / 3.6$

Reasoning check. A speed in m/s should be numerically smaller than the same speed in km/h. Always carry units through the calculation.

Worked Example 4 — Car: average speed in two units

Question. A car travels 134 km in 1.5 h. What is its average speed in km/h and in m/s?

Step-by-step solution

- Average speed = $134 \text{ km} / 1.5 \text{ h} = 89.33 \text{ km/h}$.
- Convert to metres per second: $89.33 \div 3.6 = 24.81 \text{ m/s}$.

Final answer. Average speed = $89.3 \text{ km/h} = 24.8 \text{ m/s}$.

Lesson 5 — Average velocity along a straight line

Velocity includes direction, unlike speed. **Average velocity** is displacement divided by elapsed time. If the object moves in one direction without turning around, the distance and displacement magnitudes match; then average speed and the magnitude of average velocity match too.

How the method works. A southward displacement creates a southward average velocity. If you choose north as positive, that same motion can be written with a negative sign. A negative sign on a velocity is a direction convention, not an indication that the object moves slowly.

Key equations: $v_{\text{avg}} = \Delta d / \Delta t$, including the direction of Δd

Reasoning check. State the unit (m/s or km/h) and the direction. Do not replace a direction with an unexplained negative sign.

Worked Example 5 — Cat: average velocity in one direction

Question. A cat runs 124 m south in 2.5 s. What is its average velocity?

Step-by-step solution

- Displacement = 124 m [S] .
- Average velocity = $\Delta d / \Delta t = 124 / 2.5 = 49.6 \text{ m/s}$.

Final answer. The average velocity is 49.6 m/s [S] .

Lesson 6 — Why average speed differs from average velocity

For a path with a turn, the length of the path is usually longer than the straight-line separation of its endpoints. Thus average speed and average-velocity magnitude are different quantities even though they use the same time interval.

How the method works. First calculate the **total distance** by adding each leg. Next find the **displacement vector** using geometry. Divide each by the elapsed time to obtain average speed and average velocity, respectively. The speed answer has no direction; the velocity answer does.

Key equations: Average speed = total path / Δt ; $|\mathbf{v}_{\text{avg}}| = |\text{resultant displacement}| / \Delta t$

Reasoning check. Do not use the diagonal displacement to calculate average speed. Do not use the full path distance to calculate average velocity.

Worked Example 6 — Dog: average speed versus average velocity

Question. A dog runs 240 m east, then 125 m south. The motion takes 20 s. Find the average speed and the average velocity.

Dog motion diagram



Step-by-step solution

- Total distance = 240 m + 125 m = 365 m.
- Average speed = 365 / 20 = 18.25 m/s.
- Displacement magnitude = $\sqrt{(240^2 + 125^2)} = 270.60$ m.
- Direction relative to east: $\tan \theta = 125 / 240$, so $\theta = 27.5^\circ$ south of east.
- Average velocity magnitude = 270.60 / 20 = 13.53 m/s.

Final answer. Average speed = 18.25 m/s; average velocity = 13.53 m/s [E 27.5° S].

Lesson 7 — Finding walking displacement and velocity

A walk made of north and east legs is a classic two-dimensional displacement problem. The direct shortcut from the start to the finish is the resultant displacement, not the walked distance. Two perpendicular legs allow us to use the Pythagorean theorem.

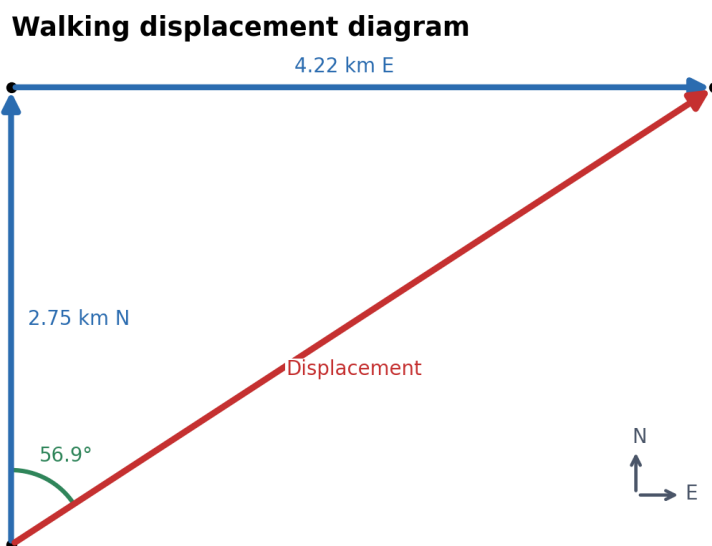
How the method works. The displacement direction comes from inverse tangent. To calculate average velocity, convert the displacement magnitude to metres and the full walking time to seconds. Velocity points in the same direction as displacement.

Key equations: $|\Delta d| = \sqrt{(\text{north}^2 + \text{east}^2)}$; $\theta = \tan^{-1}(\text{east}/\text{north})$; $|v_{\text{avg}}| = |\Delta d| / \Delta t$

Reasoning check. Keep the direction reference precise: “east of north” is not the same as “north of east.”

Worked Example 7 — Walking: displacement and average velocity

Question. You walk 2.75 km north and then 4.22 km east. The walk takes 88.5 min. Determine the displacement and the average velocity in m/s.



Step-by-step solution

- Displacement magnitude = $\sqrt{(2.75^2 + 4.22^2)} = 5.037$ km.
- Direction: $\tan \theta = 4.22 / 2.75$, so $\theta = 56.9^\circ$ east of north.
- Convert units: 5.037 km = 5,037 m and 88.5 min = 5,310 s.
- Average velocity magnitude = $5,037 / 5,310 = 0.949$ m/s.

Final answer. Displacement = 5.04 km [N 56.9° E]; average velocity = 0.949 m/s [N 56.9° E].

Concept Review — Speed and Velocity Speed describes how fast an object moves and uses the total distance travelled. It is a scalar quantity. Velocity describes how fast and in what direction an object moves. It uses displacement and is a vector quantity. Whenever the path taken is not a straight line, the distance and displacement will not be the same. As a result, average speed and the magnitude of average velocity will usually be different.

Lesson 8 — Relative velocity: two moving observers

Motion always depends on the reference point used to measure it. A passenger walking inside a train may have one velocity relative to the train and another relative to the ground. In one dimension, relative velocity can be found by subtracting the observer's velocity from the object's velocity.

How the method works. Choose right as positive (or choose left; either works if consistent). Assign signed values to each velocity. Then subtract the observer's signed velocity, remembering that subtracting a positive number makes the answer more negative.

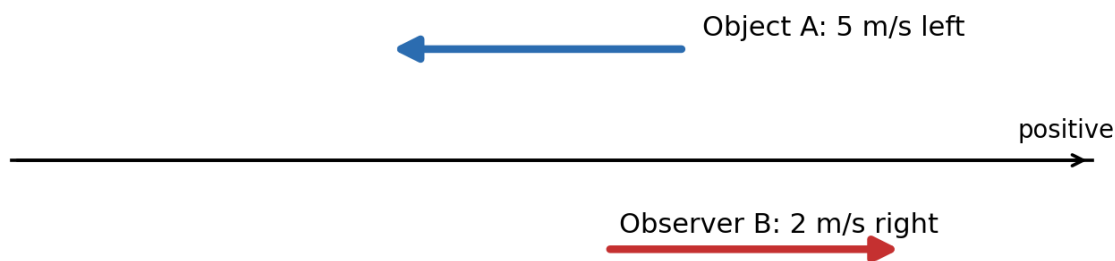
Key equations: $v_{\text{(A relative to B)}} = v_A - v_B$

Reasoning check. Interpret the sign at the end. Two objects moving apart in opposite directions can have a relative speed equal to the sum of their speed magnitudes.

Worked Example 8 — Relative motion in one dimension

Question. An object moves at 5 m/s to the left while an observer moves at 2 m/s to the right. What velocity does the observer measure for the object?

One-dimensional relative motion



$$v_{A \text{ relative to } B} = v_A - v_B = (-5) - (+2) = -7 \text{ m/s}$$

So A is seen as moving 7 m/s to the left.

Step-by-step solution

- Object velocity: $v_A = -5 \text{ m/s}$.
- Observer velocity: $v_B = +2 \text{ m/s}$.
- Relative velocity: $v_{A \text{ relative to } B} = v_A - v_B = -5 - (+2) = -7 \text{ m/s}$.
- The negative sign means the motion is to the left.

Final answer. The observer measures the object moving at 7 m/s left.

Lesson 9 — Position, displacement, and the origin

A position gives a location relative to an agreed origin, such as kilometre zero on a highway. Positions on the left or west side can be assigned negative signs; positions to the right or east are positive.

How the method works. Displacement is **final position minus initial position**. Subtract the initial signed value, including its minus sign. If the object starts west of zero and finishes east of zero, the displacement includes the full distance on both sides of the origin. Convert minutes to hours before obtaining km/h.

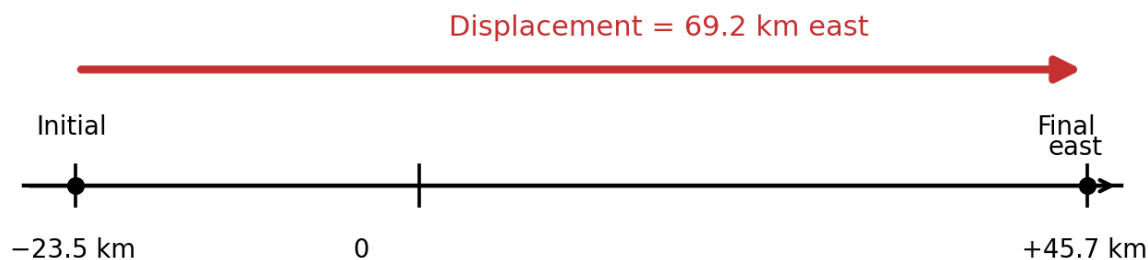
Key equations: $\Delta x = x_f - x_i$; $v_{\text{avg}} = \Delta x / \Delta t$

Reasoning check. Use parentheses around a negative initial position. For example, subtracting -23.5 means adding 23.5.

Worked Example 9 — Average velocity from initial and final position

Question. An object starts at 23.5 km west of the origin and ends at 45.7 km east of the origin after 75 min. What is its average velocity in km/h?

Motion on a one-dimensional position axis



Step-by-step solution

- Initial position: $x_i = -23.5$ km.
- Final position: $x_f = +45.7$ km.
- Displacement: $\Delta d = x_f - x_i = 45.7 - (-23.5) = 69.2$ km east.
- Elapsed time: 75 min = 1.25 h.
- Average velocity = $69.2 / 1.25 = 55.36$ km/h.

Final answer. The average velocity is 55.4 km/h [E].

Lesson 10 — Upward motion and the meaning of gravity

A vertically thrown ball slows down as it rises because Earth's gravitational acceleration always points downward. Ignoring air resistance, that downward acceleration is approximately 9.81 m/s^2 . The ball's upward velocity gets smaller every second until it becomes zero at the peak.

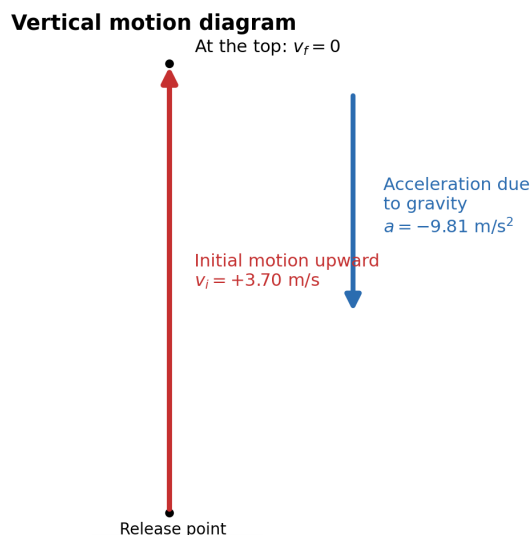
How the method works. The highest point is marked by $v_f = 0$, but the acceleration is still -9.81 m/s^2 if upward is positive. The velocity equation tells us how long gravity takes to change the upward launch speed to zero.

Key equations: $v_f = v_i + a\Delta t$; $\Delta t = (v_f - v_i)/a$

Reasoning check. Velocity is zero for an instant at the top; gravity does not switch off there.

Worked Example 10 — Ball thrown straight upward

Question. A ball is thrown vertically upward with an initial velocity of 3.70 m/s. How long does it take to reach maximum height?



Step-by-step solution

- Given: $v_i = +3.70 \text{ m/s}$, $v_f = 0$, and $a = -9.81 \text{ m/s}^2$. v_i v_f
- Use the kinematics equation $v_f = v_i + a\Delta t$.
- Rearrange for time: $\Delta t = (v_f - v_i) / a$.
- Substitute values: $\Delta t = (0 - 3.70) / (-9.81) = 0.377 \text{ s}$.

Final answer. The ball takes 0.377 s to reach maximum height.

Lesson 11 — Finding position from uniform velocity

Uniform motion means velocity is constant in both magnitude and direction. In equal time intervals, the moving object makes equal displacements. If velocity remains constant, the distance from the starting point grows proportionally with elapsed time.

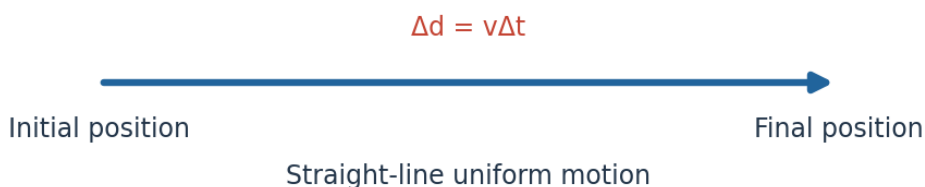
How the method works. Multiply velocity by time to find displacement. Then add displacement to the starting position to find the final position. If the object begins at the origin, its final position is the same as its displacement. Units cancel correctly: $(\text{m/s}) \times \text{s} = \text{m}$.

Key equations: $\Delta d = v\Delta t$; $d_f = d_i + \Delta d$

Reasoning check. Distinguish the origin (position zero) from an object being at rest (velocity zero).

Worked Example 11 — Finding final position

Question. An object starts at the origin and travels north at a constant velocity of 3.2 m/s for 13 s. What is its final position?



Step-by-step solution

- Take north as positive and set $d_i = 0$.
- $\Delta d = (3.2 \text{ m/s})(13 \text{ s}) = 41.6 \text{ m north}$.
- $d_f = 0 + 41.6 \text{ m} = 41.6 \text{ m north of the origin}$.

Final answer. 41.6 m [N].

Lesson 12 — Displacement divided by time

Average velocity tells how rapidly position changes over a measured interval. In a straight-line journey with one direction, the displacement magnitude is the route length and the velocity direction matches the travel direction.

How the method works. Choose a sign convention or write compass directions explicitly. Divide the displacement by time, then attach its direction. Unlike average speed, average velocity may be negative in a signed-axis calculation.

Key equations: $v_{\text{avg}} = (d_f - d_i)/\Delta t = \Delta d/\Delta t$

Reasoning check. A westward velocity is a vector. The same magnitude without “west” would describe only a speed.

Worked Example 12 — Finding velocity from displacement

Question. An object travels 113 km west in 2.3 h. What is its average velocity?

Step-by-step solution

- $v_{\text{avg}} = \Delta d/\Delta t = 113 \text{ km} / 2.3 \text{ h} = 49.13 \text{ km/h}$.

Final answer. 49.1 km/h [W].

Lesson 13 — Maximum height from a launched position

For a ball thrown upward from above the ground, there are **two heights**: the height it gains after release, and its total height above the ground. Gravity slows the ball until vertical speed is zero at the top.

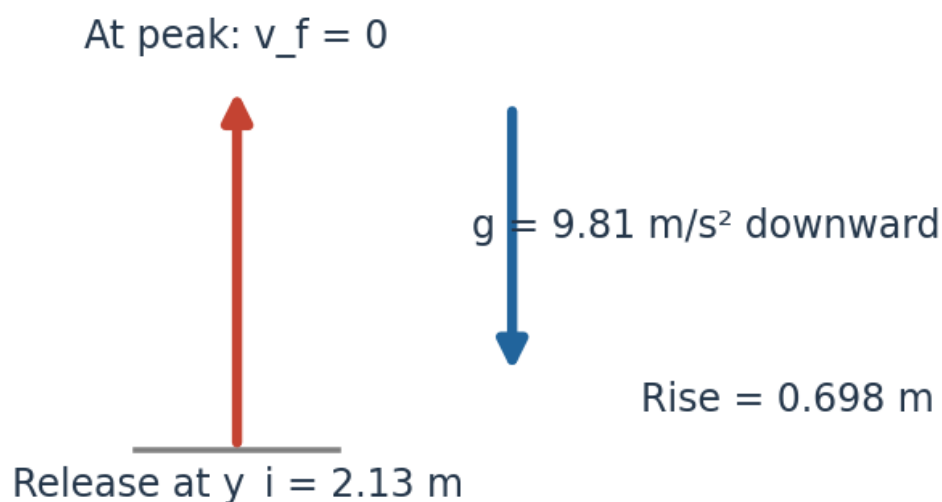
How the method works. The equation linking velocity, acceleration, and displacement works even when time is not provided. Set final velocity to zero at the peak, solve for the upward rise, then add the initial release height.

Key equations: $v_f^2 = v_i^2 + 2a\Delta y$; $h_{\text{max}} = h_{\text{release}} + \Delta y$

Reasoning check. Do not report only the extra rise if the question asks for height above the ground.

Worked Example 13 — Maximum height of a thrown ball

Question. A ball is launched vertically upward at 3.70 m/s from a height of 2.13 m above the ground. What maximum height above the ground does it reach?



Step-by-step solution

- Use $v_f^2 = v_i^2 + 2a\Delta y$. $v_f = 0$
- At the top, $v_f = 0$, $v_i = +3.70 \text{ m/s}$, $a = -9.81 \text{ m/s}^2$. $v_f^2 = v_i^2 + 2a\Delta y$
- $\Delta y = (0 - 3.70^2)/(2 \times -9.81) = 0.698 \text{ m}$.
- Maximum height = $2.13 + 0.698 = 2.828 \text{ m}$.

Final answer. 2.83 m above the ground.

Lesson 14 — Objects dropped from rest

A dropped ball starts with zero vertical speed, then gains downward speed because of gravity. In ideal free fall, gravity causes an almost constant acceleration. A convenient convention is to call downward positive, making $a = +9.81 \text{ m/s}^2$.

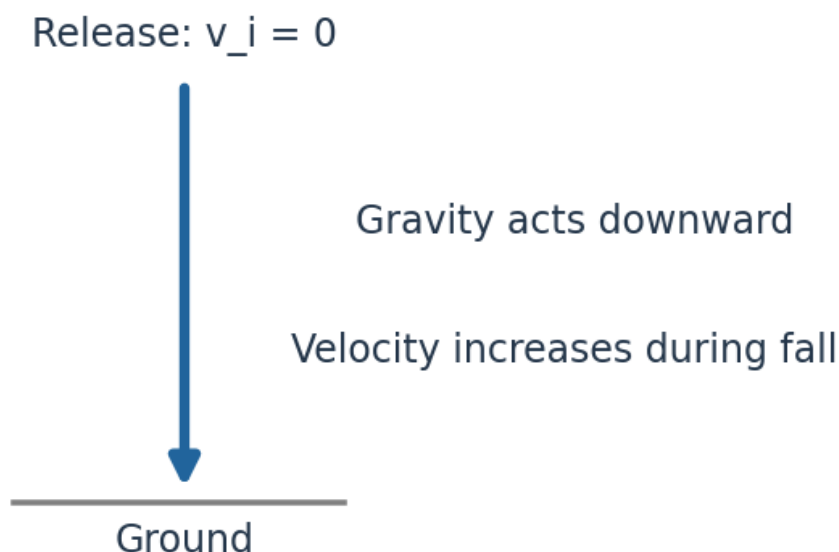
How the method works. To determine the impact velocity, multiply acceleration by time because initial velocity is zero. To determine displacement, use the constant-acceleration position equation. Squaring time means a longer fall produces much more than a proportional increase in distance.

Key equations: $v_f = v_i + a\Delta t$; $\Delta y = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$

Reasoning check. Choose your positive direction and stick with it in every equation.

Worked Example 14 — Dropped ball: impact speed and distance

Question. A ball is dropped from rest and strikes the ground 5.82 s later. Find (a) its impact velocity and (b) its distance fallen. Neglect air resistance.



Step-by-step solution

- Taking downward as positive, $v_i = 0$ and $a = +9.81 \text{ m/s}^2$. v_i
- $v_f = v_i + a\Delta t = (9.81)(5.82) = 57.09 \text{ m/s}$.
- $\Delta y = +\frac{1}{2}a(\Delta t)^2 = \frac{1}{2}(9.81)(5.82)^2 = 166.17 \text{ m}$. $v_i\Delta t$

Final answer. Impact velocity 57.1 m/s downward; distance fallen 166 m.

Lesson 15 — Working backward to an initial velocity

Kinematics formulas can be rearranged to solve for any unknown, not just time or final velocity. If an object accelerates downward after being thrown, its final speed reflects both the initial throw and the extra speed gained under gravity.

How the method works. Start with the familiar relationship between final velocity, initial velocity, acceleration, and time. Rearrange to isolate initial velocity. With downward positive, both a downward final velocity and gravity are positive.

Key equations: $v_i = v_f - a\Delta t$

Reasoning check. Check that the initial downward speed is smaller than the final downward speed when gravity speeds the object up.

Worked Example 15 — Thrown downward: finding initial velocity

Question. A ball is thrown straight down and reaches the ground 1.13 s later with velocity 13.7 m/s downward. What was its initial velocity?

Step-by-step solution

- Take downward as positive. $= +13.7 \text{ m/s}$; $a = +9.81 \text{ m/s}^2$. v_f
- $v_i = v_f - a\Delta t = 13.7 - (9.81)(1.13) = 2.6147 \text{ m/s}$.

Final answer. 2.61 m/s downward.

Lesson 16 — Wind as an added velocity vector

An animal flying through air moves relative to the air. But the air itself can move relative to the ground. To determine the flight path seen by someone standing on Earth, add the flying velocity and the wind velocity as vectors.

How the method works. If the two motions are perpendicular, one is a horizontal component and the other is a vertical component. Draw an arrow for each tip-to-tail, then join start to finish with the resultant. Use Pythagoras for speed and tangent for direction.

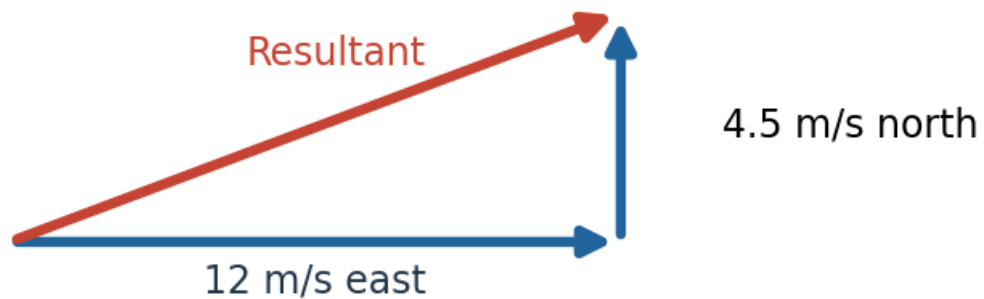
Key equations: $v_{\text{ground}} = v_{\text{relative-to-air}} + v_{\text{wind}}$; $|v| = \sqrt{v_x^2 + v_y^2}$

Reasoning check. Wind direction is the direction the air actually moves. Check whether the wording says “wind toward north” or “wind from north.”

Worked Example 16 — Bird flying in wind

Question. A bird flies due east at 12 m/s relative to the air while a northerly-directed wind moves at 4.5 m/s. Find the bird's ground velocity.

N ↑ E →

**Step-by-step solution**

- East component: 12 m/s; north component: 4.5 m/s.
- Magnitude = $\sqrt{(12^2 + 4.5^2)} = 12.82$ m/s.
- Direction east of north: $\theta = \tan^{-1}(12/4.5) = 69.4^\circ$.

Final answer. 12.8 m/s [N 69.4° E] (or [E 20.6° N]).

Lesson 17 — Swimming and moving water

A fish can aim one way but travel diagonally across the ground because the water carries it sideways. This is relative motion in two dimensions. The fish's velocity relative to water and the water's velocity relative to the riverbank must be added.

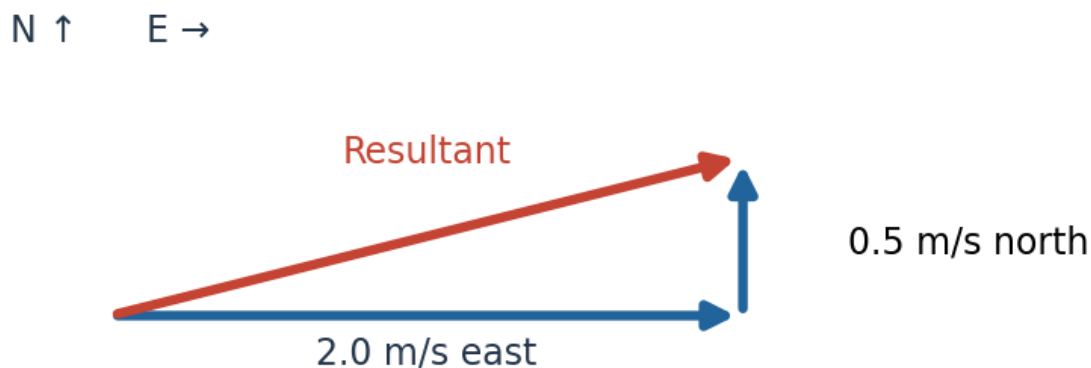
How the method works. Use independent north and east axes. Even if one component is much larger, both must appear in the final ground velocity. A compass angle tells whether the actual motion is mostly eastward or mostly northward.

Key equations: $\mathbf{v}_{\text{fish,ground}} = \mathbf{v}_{\text{fish,water}} + \mathbf{v}_{\text{water,ground}}$

Reasoning check. If the eastward current is stronger than the northward swimming, the resultant points closer to east than to north.

Worked Example 17 — Fish swimming across a river

Question. A fish swims north at 0.5 m/s relative to the water, while river water flows east at 2.0 m/s. Determine the fish's velocity relative to the ground.

**Step-by-step solution**

- North component = 0.5 m/s; east component = 2.0 m/s.
- Magnitude = $\sqrt{(0.5^2 + 2.0^2)} = 2.0616$ m/s.
- Direction east of north: $\theta = \tan^{-1}(2.0/0.5) = 75.96^\circ$.

Final answer. 2.06 m/s [N 76.0° E].

Lesson 18 — Horizontal projectiles: shared flight time

When a ball rolls off a level tabletop, it leaves with horizontal velocity but no initial vertical velocity. Gravity immediately gives it downward acceleration. The ball follows a curved trajectory because horizontal and vertical motions occur at the same time.

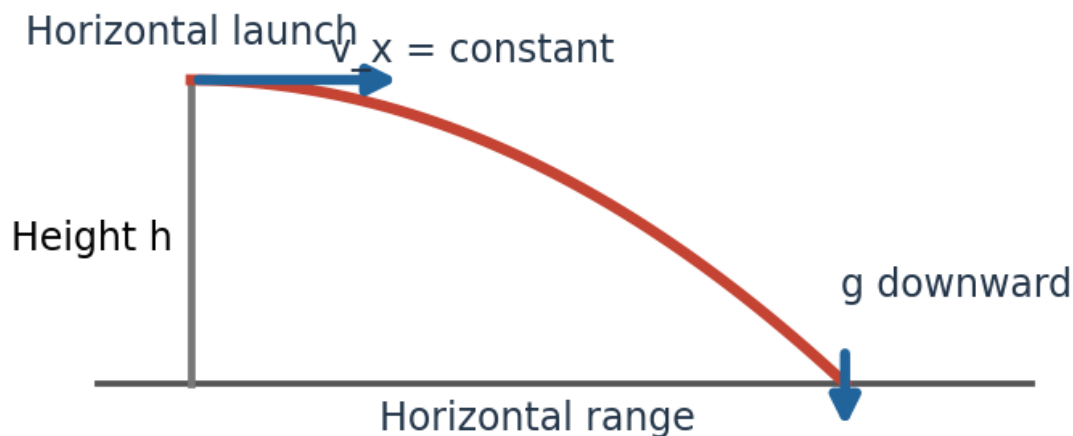
How the method works. Solve the vertical problem first to find the flight time, using the drop height. Then use that same time in the horizontal motion. At impact, the horizontal speed is unchanged (ignoring air resistance) while the downward speed has increased. Combine these two velocity components to obtain impact speed and direction.

Key equations: $h = \frac{1}{2}gt^2$; $t = \sqrt{(2h/g)}$; $\Delta x = v_x t$; $|v_{\text{impact}}| = \sqrt{(v_x^2 + v_y^2)}$

Reasoning check. Use height to calculate time, not the launch speed; label positive and negative directions consistently.

Worked Example 18 — Horizontal launch: 0.91 m drop

Question. A ball leaves the edge of a table horizontally at 2.0 m/s. The ground is 0.91 m below the launch point. Find the flight time, horizontal range, and impact velocity.

**Step-by-step solution**

- Vertical: $0.91 = \frac{1}{2}(9.81)t^2$, so $t = \sqrt{(2 \times 0.91/9.81)} = 0.431 \text{ s}$.
- Horizontal range: $\Delta x = (2.0)(0.431) = 0.861 \text{ m}$.
- At impact, $v_{x,f} = 2.0 \text{ m/s}$ and $v_{y,f} = -(9.81)(0.431) = -4.23 \text{ m/s}$.
- Impact speed = $\sqrt{(2.0^2 + 4.23^2)} = 4.68 \text{ m/s}$.
- Angle below horizontal = $\tan^{-1}(4.23/2.0) = 64.7^\circ$.

Final answer. Flight time 0.431 s; range 0.861 m; impact velocity 4.68 m/s, 64.7° below the horizontal.

Lesson 19 — How horizontal launch speed changes range

Two balls launched horizontally from the same height reach the ground at the same time when air resistance is ignored, even if one travels faster sideways. Their vertical motions are identical because both begin with zero vertical velocity and have the same acceleration.

How the method works. First find time from the table height. Then multiply that time by the new ball's horizontal speed to find its range. Increasing horizontal speed increases range but does not change how long gravity needs to produce the given vertical drop.

Key equations: $t = \sqrt{(2h/g)}$; range = horizontal speed $\times t$

Reasoning check. Time depends on the fall height; horizontal range also depends on the horizontal speed.

Worked Example 19 — Faster horizontal launch from the same table

Question. A second ball rolls horizontally off the same 0.91 m high table with speed 3.55 m/s. How long is it airborne and how far from the table does it land?

Step-by-step solution

- Time remains $t = \sqrt{(2 \times 0.91/9.81)} = 0.431 \text{ s}$.
- Horizontal distance = $(3.55 \text{ m/s})(0.431 \text{ s}) = 1.53 \text{ m}$.

Final answer. Time 0.431 s; horizontal range 1.53 m.

Lesson 20 — Crossing a river: time and drift

A swimmer's movement across a river can be separated into two independent components. The swimmer creates a velocity across the width, while the moving current produces drift downstream. The ground sees the combined diagonal motion.

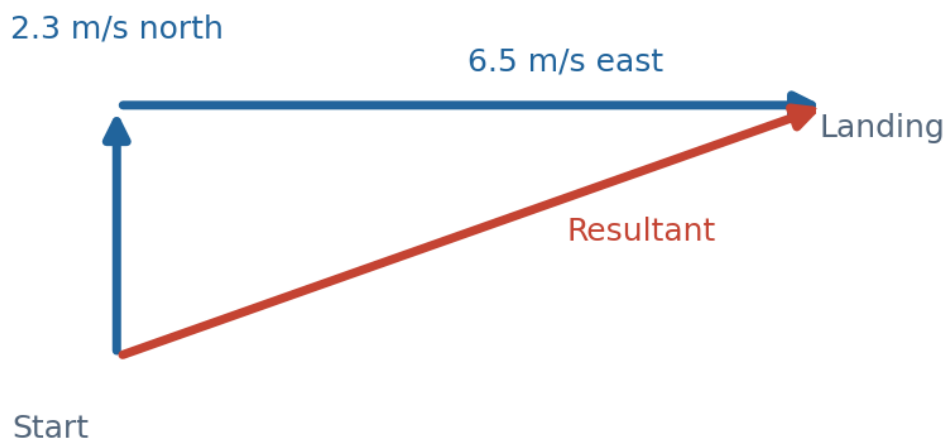
How the method works. The key is to use the component perpendicular to the banks when calculating how long it takes to cross. After finding that time, multiply by the downstream current speed to find drift. Finally, combine the two velocity components to describe the direction of travel.

Key equations: $t_{\text{cross}} = \text{river width} / v_{\text{across}}$; $\text{drift} = v_{\text{current}} \times t_{\text{cross}}$

Reasoning check. A fast current changes landing position and ground-speed magnitude, but not the crossing time if the across-river swimming component stays the same.

Worked Example 20 — Swimming across a river

Question. Marcel swims due north at 2.3 m/s relative to the water to cross a river 255 m wide. The current flows east at 6.5 m/s . (a) How long does it take to cross? (b) How far downstream does Marcel arrive? (c) What is his velocity relative to the ground?



Step-by-step solution

- Time: $\Delta t = 255 / 2.3 = 110.87 \text{ s}$.
- Eastward drift: $\Delta x = (6.5)(110.87) = 720.65 \text{ m east}$.
- Ground velocity components: $v_x = 6.5 \text{ m/s east}$, $v_y = 2.3 \text{ m/s north}$.

- Magnitude = $\sqrt{(6.5^2 + 2.3^2)} = 6.89 \text{ m/s}$.
- Direction = $\tan^{-1}(6.5 / 2.3) = 70.5^\circ$ east of north.

Final answer. Crossing time 111 s; drift 721 m east; ground velocity 6.89 m/s [N 70.5° E].

Lesson 21 — Crosswinds during a straight crossing

A person crossing an opening while wind blows sideways follows a diagonal path relative to the ground. The forward component carries them across the gap; the wind component carries them sideways. These effects occur simultaneously.

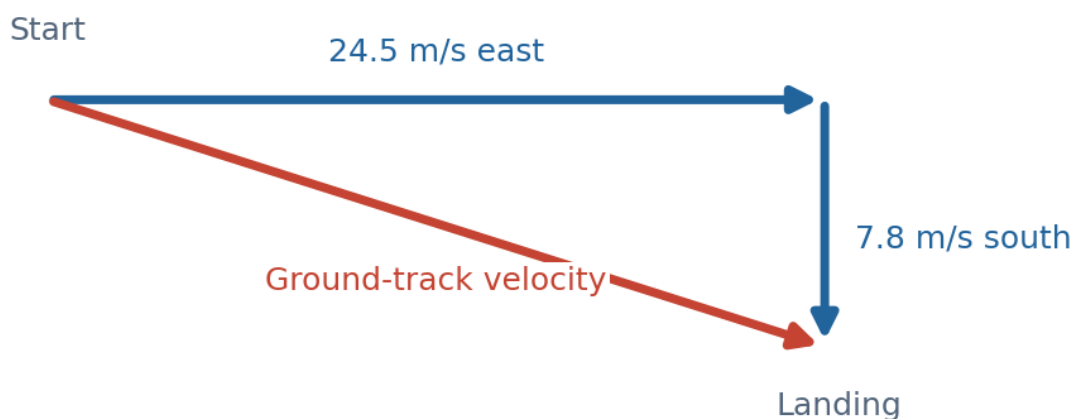
How the method works. For a simple constant-velocity model, first add perpendicular velocity components to obtain ground velocity. Then divide the crossing width by the forward component, not by the total ground speed. Finally, multiply the wind component by the crossing time to determine sideways drift.

Key equations: $\mathbf{v}_{\text{ground}} = (\mathbf{v}_{\text{forward}}, \mathbf{v}_{\text{side}})$; $t = \text{width} / v_{\text{forward}}$; $\text{drift} = v_{\text{side}} t$

Reasoning check. This is an idealized motion model. Real flight also involves gravity, air resistance, and control forces.

Worked Example 21 — Wingsuit crossing in a crosswind

Question. Zak travels east at 24.5 m/s relative to the air while crossing a 133 m wide gorge. The wind blows south at 7.8 m/s. Assuming these velocities remain constant, determine (a) his velocity relative to the ground, (b) crossing time, and (c) southward drift.



Step-by-step solution

- Ground velocity components are $v_x = 24.5 \text{ m/s}$ east and $v_y = 7.8 \text{ m/s}$ south.
- Magnitude = $\sqrt{(24.5^2 + 7.8^2)} = 25.71 \text{ m/s}$.
- Direction = $\tan^{-1}(7.8 / 24.5) = 17.7^\circ$ south of east.

- Crossing time = $133 / 24.5 = 5.43$ s.
- Southward drift = $(7.8)(5.43) = 42.3$ m south.

Final answer. Ground velocity 25.7 m/s [E 17.7° S]; crossing time 5.43 s; drift 42.3 m south.

Lesson 22 — Relative velocity in the same direction

Relative motion is especially straightforward when both velocities lie on the same vertical line. A basket lowered relative to an upward-rising balloon may still travel upward relative to the ground. Different observers measure different velocities.

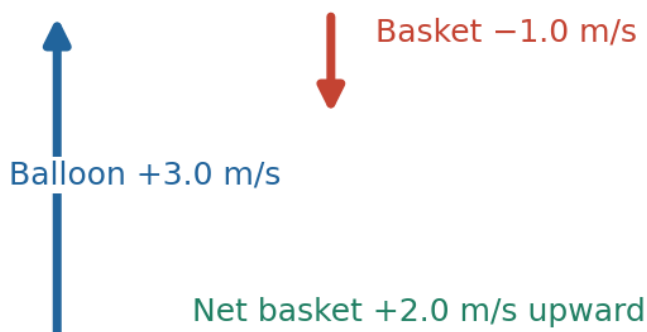
How the method works. Set upward as positive. Write the balloon's velocity relative to the ground and the basket's velocity relative to the balloon, assigning negative signs to downward quantities. Add them algebraically to obtain basket velocity relative to ground.

Key equations: $v_{\text{basket,ground}} = v_{\text{basket,balloon}} + v_{\text{balloon,ground}}$

Reasoning check. Avoid saying an object moves downward without specifying what it is moving downward relative to.

Worked Example 22 — Basket descending within a rising balloon

Question. A hot-air balloon rises at 3.0 m/s relative to the ground. A passenger lowers a basket at 1.0 m/s relative to the balloon. What is the basket's velocity relative to the ground?



Step-by-step solution

- Take upward as positive: $v_{\text{balloon,ground}} = +3.0$ m/s and $v_{\text{basket,balloon}} = -1.0$ m/s.
- $v_{\text{basket,ground}} = +3.0 + (-1.0) = +2.0$ m/s.

Final answer. The basket moves 2.0 m/s upward relative to the ground.

Lesson 23 — Airplane ground speed and headwinds

An airplane moves through the surrounding air at its airspeed. Wind moves that air relative to the ground. Thus the ground speed may be smaller or greater than airspeed depending on the wind direction.

How the method works. A headwind points opposite the plane's route, so subtract the wind speed from the airspeed to find ground speed. Then use distance divided by ground speed to calculate trip time. For the extra time lost, calculate a second trip time without wind and subtract it.

Key equations: $v_{\text{ground}} = v_{\text{airplane,air}} - v_{\text{headwind}}$; $t = \text{route distance} / \text{ground speed}$

Reasoning check. Keep all speeds in km/h when the route distance is in kilometres; the resulting time is in hours.

Worked Example 23 — Airplane facing a headwind

Question. An airplane flies west at 850 km/h relative to the air. A jet stream blows east at 177 km/h. (a) What is the airplane's ground velocity? (b) How long does it take to cover a 3,359 km westward route? (c) How much travel time does the headwind add compared with still air?



Step-by-step solution

- Take west positive: $v_{\text{ground}} = 850 - 177 = 673 \text{ km/h west}$.
- Time with wind: $t_{\text{wind}} = 3359 / 673 = 4.991 \text{ h}$.
- Time without wind: $t_{\text{still}} = 3359 / 850 = 3.952 \text{ h}$.
- Time lost = $4.991 - 3.952 = 1.039 \text{ h} \approx 62.4 \text{ min}$.

Final answer. Ground velocity 673 km/h [W]; travel time 4.99 h; headwind adds about 1.04 h (62 min).

Lesson 24 — Angled projectiles: maximum height, time, and range

Launching a ball at an angle splits its starting velocity into horizontal and vertical parts. The horizontal velocity remains constant if air resistance is negligible. The vertical velocity changes under gravity: it decreases on ascent and becomes negative on descent.

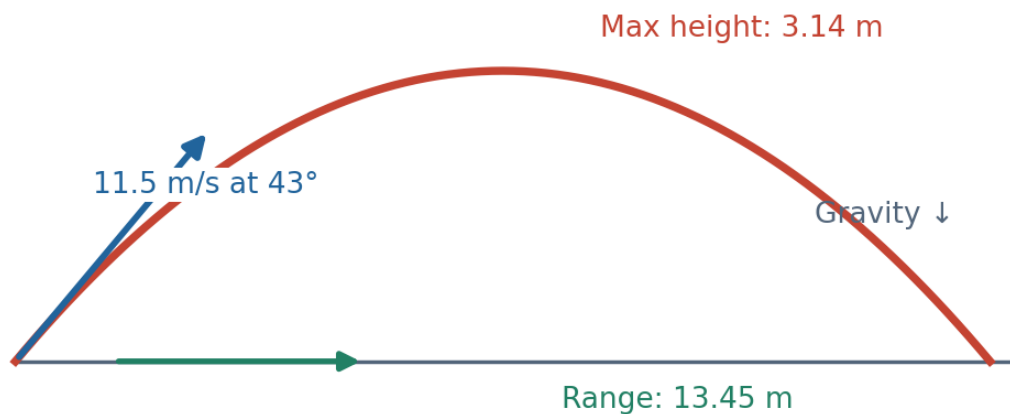
How the method works. First resolve initial velocity using cosine and sine. At the highest point, vertical velocity is zero, so time to peak equals vertical launch speed divided by g . If the ball lands at the same height from which it started, total flight time is twice the time to peak. Use the flight time and horizontal component to find range.

Key equations: $v_{ix} = v_i \cos \theta$; $v_{iy} = v_i \sin \theta$; $t_{\text{flight}} = 2v_{iy}/g$; $h_{\text{max}} = v_{iy}^2/(2g)$; $R = v_{ix} t$

Reasoning check. These simplified total-time and range formulas assume launch and landing at the same elevation.

Worked Example 24 — Soccer ball launched at 43°

Question. A soccer ball is kicked at 11.5 m/s at 43° above horizontal. Assume it lands at its launch elevation and air resistance is negligible. Find its maximum height, total time in the air, and horizontal range.



Step-by-step solution

- $v_{ix} = 11.5 \cos 43^\circ = 8.41 \text{ m/s}$; $v_{iy} = 11.5 \sin 43^\circ = 7.84 \text{ m/s}$.
- Time to peak = $7.84 / 9.81 = 0.799 \text{ s}$.
- Flight time = $2(7.84)/9.81 = 1.60 \text{ s}$.
- Maximum height = $7.84^2/(2 \times 9.81) = 3.14 \text{ m}$.
- Horizontal range = $(8.41)(1.599) = 13.45 \text{ m}$.

Final answer. Maximum height 3.14 m; time in air 1.60 s; horizontal range 13.4 m.

Lesson 25 — Angle, initial speed, and projectile trajectory

A greater starting speed produces both stronger horizontal motion and stronger vertical motion. But the launch angle decides how that speed is divided. A steeper angle gives more upward velocity and generally more airtime, while a shallower angle gives more immediate horizontal speed.

How the method works. Solve the vertical and horizontal parts separately while using the same time interval. For level-ground launch and landing, vertical displacement over the entire flight is zero. Ignore the trivial time $t =$

0 (the launch moment) and solve for the nonzero return time. Use this to determine maximum height and range.

Key equations: $0 = v_{iy} t - \frac{1}{2}gt^2$; $t_{\text{flight}} = 2v_{iy}/g$; $h_{\text{max}} = v_{iy}^2/(2g)$; $R = v_{ix} t_{\text{flight}}$

Reasoning check. At the highest point the vertical velocity is zero, but the horizontal velocity is still nonzero.

Worked Example 25 — Soccer ball launched at 33°

Question. A ball is kicked from level ground at 23 m/s and 33° above horizontal. Determine the total flight time, maximum height, and horizontal range. Ignore air resistance.



Step-by-step solution

- $v_{ix} = 23 \cos 33^\circ = 19.29 \text{ m/s}$; $v_{iy} = 23 \sin 33^\circ = 12.53 \text{ m/s}$.
- Flight time: $2(12.53)/9.81 = 2.55 \text{ s}$.
- Maximum height: $12.53^2/(2 \times 9.81) = 8.00 \text{ m}$.
- Range: $(19.29)(2.5539) = 49.26 \text{ m}$.

Final answer. Flight time 2.55 s ; maximum height 8.00 m ; horizontal range 49.3 m .